UDC 62-50

LINEAR DIFFERENTIAL ENCOUNTER GAME WITH A FUNCTIONAL TARGET

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For a conflict-controlled system described by ordinary linear differential equations we study the problem of constructing a control method ensuring under conditions of uncertainty the existence of specified properties on a certain segment of the system's trajectory. We indicate sufficient conditions for the solvability of the problem and a method for constructing the resolving controls. The paper is related to the investigations in [1—7].

1. We consider the controlled system

$$x^{\cdot}(t) = A(t)\, x(t) + B(t)\, u - C(t)\, v + w(t), \quad t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$
$$u \in P(t) \subset E_{r_1}, \quad v \in Q(t) \subset E_{r_2}$$

Here x is the n-dimensional phase coordinate vector; vectors u and v are the controls of the first and second players, respectively; $P(t)$ and $Q(t)$ are convex compacta continuous in t; matrices $A(t)$, $B(t)$ and $C(t)$ are continuous; $w(t)$ is a given perturbation (a Lebesgue-integrable function). The initial state $x(t_0) = x_0$, a finite instant ϑ and a number $\tau \in [0, \vartheta - t_0]$ are given. A certain family of functions N is defined on the interval $[\vartheta - \tau, \vartheta]$. The first player strives to ensure the inclusion $x_\vartheta(s) \in N$ for the trajectory segment $x_\vartheta(s) = x(\vartheta + s)$, $-\tau \leqslant s \leqslant 0$. At each instant $t_* \in [t_0, \vartheta - \tau]$ he knows the vector $x(t_*)$, while for $t_* \in (\vartheta - \tau, \vartheta]$ he knows the system's previous history $x(t_* + s)$, $s \in [\vartheta - \tau - t_*, 0]$. The second player's purpose is the opposite and the information available to him can be arbitrarily complete; he can select any method for forming the control v developing a measurable realization $v[t]$.

Following [3, 4], the segment $x_t(s) = x(t + s)$, $s \in [-\tau, 0]$ of the trajectory $x(t)$ is called a state of system (1.1) at the instant t (we assume $x_{t_0}(s) \equiv 0$, $s \in [-\tau, 0)$); the pair $\{t, x(s)\}$, $t \in [t_0, \vartheta]$, $x(s) \in H$, where H is a Hilbert space of functions $x(s)$ with the norm

$$\| x(s) \|_\tau = \left(\| x(0) \| + \int_{-\tau}^{0} \| x(s) \|^2 ds \right)^{1/2}$$
$$\| z \| = (z_1^2 + z_2^2 + \ldots + z_n^2)^{1/2}, \quad z \in E_n$$

is called a position p . The rule which associates with each position p a set $U(p) \subset P(t)$, convex, closed and upper-semicontinuous in t and x, where t varies from the right, is called the first player's strategy U. Any function $x[t] = x[t; p_*, U]$, $t_* - \tau \leqslant t \leqslant \vartheta$, absolutely continuous on $[t_*, \vartheta]$, satisfying the condition $x[t_* + s] = x^*(s)$ and, for almost all $t \in [t_*, \vartheta]$, the equality

$$x^{\cdot}[t] = A(t)\, x[t] + B(t)\, u[t] - C(t)\, v[t] + w(t) \tag{1.2}$$

where the measurable functions $u\,[t]$ and $v\,[t]$ satisfy the inclusion

$$u\,[t] \in U\,(t,\ x_t\,[s]), \quad v\,[t] \in Q\,(t) \tag{1.3}$$

for almost all t, is called a motion of system (1.1) from the position $p_* = \{t_*,\ x^*\,(s)\}$, $t_* \geqslant t_0$, corresponding to strategy U . A motion defined in such a way exists (*).

Problem 1. Given system (1.1), a position $p_0 = \{t_0,\ x_0\,(s)\}$ an instant $\vartheta \geqslant t_0 + \tau$ and a closed, convex and bounded set $N \subset H$. Construct an admissible strategy $U_e\,(p)$ such that all motions $x\,[t] = x\,[t;\ p_0,\ U_e]$ satisfy the inclusion $x_\vartheta\,[s] \in N$.

Problem 2. Construct an admissible strategy $U_*\,(p)$ solving Problem 1 and satisfying the conditions:

1) for $t \in [t_0,\ \vartheta - \tau]$ $U_*\,(t,\ x\,(s)) = U_*\,(t,\ y\,(s))$,
if $x\,(0) = y\,(0)$;

2) for $t \in (\vartheta - \tau,\ \vartheta]$ $U_*\,(t,\ x\,(s)) = U_*\,(t,\ y\,(s))$, if $x\,(s) = y\,(s)$, $\vartheta - \tau - t \leqslant s \leqslant 0$.

for any elements $x\,(s) \in H$, $y\,(s) \in H$.

Problem 2 can obviously be treated as a mathematical formalization of the original control problem. Problem 1 was studied in [5] for a linear controlled system with time lag. It is easy to see that the method presented in [5] for solving this problem from the standpoint of the extremal aiming principle [3, 4] applied to system (1.1) also solves Problem 2. The structure of system (1.1) permits us to make concrete (in comparison with [5]) the results obtained in this way and to find effective sufficient conditions for the solvability of Problem 2 and a convenient method for constructing the resolving strategy. These questions are discussed in the present paper.

Let $t_* \in [t_0,\ \vartheta]$ and $x_*\,(s) \in H$ be given and let $u\,(t)$ and $v\,(t)$ be some programs as functions measurable on $[t_0,\ \vartheta]$ for almost all t, with values in $P\,(t)$ and $Q\,(t)$, respectively. Let $x\,(t) = x\,(t;\ \{t_*,\ x_*\,(s)\},\ u,\ v)$, $t_* - \tau \leqslant t \leqslant \vartheta$ be a function satisfying the conditions $x\,(t_* + s) = x_*\,(s)$; $x\,(t)$, $t_* \leqslant t \leqslant \vartheta$ is an absolutely continuous solution of Eq. (1.1) when $u = u\,(t)$ and $v = v\,(t)$. By the symbol $W\,(t,\ N)$ we denote the collection of all $x\,(s) \in H$ with the property: for any program $v\,(t)$ there exists a program $u\,(t)$ such that $x\,(\vartheta + s;\ \{t_*,\ x\,(s)\},\ u,\ v) \in N$ ($W\,(t\ ,\ N)$ is the analog of the program absorption sets in [1—5]).

Let $B\,(t,\ \vartheta)$ be an operator acting from H into H and defined as follows:

1) if $t \leqslant \vartheta - \tau$, then

$$B(t,\vartheta)h = h_1 = \begin{cases} h'(0)\,X(\vartheta,t) + \int\limits_{-\tau}^{0} h'(\xi)\,X(\vartheta+\xi,t)\,d\xi, & s = 0 \\ 0, \quad s \in [-\tau, 0) \end{cases}$$

*) Questions on the existence of solutions of differential inclusions with aftereffect and with initial functions from space H were examined in: Osipov, Iu. S., Problems in the Theory of Differential-Difference Games, Sverdlovsk, Doctoral Dissertation, 1971. The existence theorem for the inclusions in (1.3) was proved therein.

2) if $t \in (\vartheta - \tau, \vartheta]$, then $(\delta = \vartheta - t)$

$$B(t,\vartheta)h = h_1 = \begin{cases} h'(0)X(\vartheta,t) + \int\limits_{-\delta}^{0} h'(\xi)X(\vartheta+\xi,t)\,d\xi, & s = 0 \\ h(s-\delta), \quad s \in [-\tau+\delta, 0) \\ 0, \quad s \in [-\tau, -\tau+\delta) \end{cases}$$

Here $X(\vartheta, t)$ is the transition matrix of the system $\dot{x}(t) = A(t)x(t)$; the prime denotes transposition. We consider the functional $\alpha_t(\xi, h)$:

1) if $t < \vartheta - \tau$, then

$$\alpha_t(\xi,h) = \begin{cases} \int\limits_{-\tau}^{0} h'(s)X(\vartheta+s,\xi)\,ds, & \xi \in [t, \vartheta-\tau) \\ \int\limits_{\xi-\vartheta}^{0} h'(s)X(\vartheta+s,\xi)\,ds, & \xi \in [\vartheta-\tau, \vartheta] \end{cases}$$

2) if $t \in [\vartheta - \tau, \vartheta]$, then

$$\alpha_t(\xi,h) = \int\limits_{\xi-\vartheta}^{0} h'(s)X(\vartheta+s,\xi)\,ds, \quad \xi \in [t,\vartheta]$$

The following assertion, arising from the theorem on the separability of convex sets in H, is valid.

Theorem 1.1. $x(s) \in W(t, N)$ if and only if

$$\gamma(\vartheta,t,x) = \max_{\|h\|_\tau \leqslant 1} \{-\varphi(t,\vartheta,h,x)\} \leqslant 0 \tag{1.4}$$

Here

$$\varphi(t,\vartheta,h,x) = \rho(\vartheta,t,h) + \langle B(t,\vartheta)h, x\rangle$$

$$\rho(\vartheta,t,h) = \int\limits_t^\vartheta \rho_u(t,\xi,h)\,d\xi - \int\limits_t^\vartheta \rho_v(t,\xi,h)\,d\xi - \rho_N(h) + \int\limits_t^\vartheta \{h'(0)X(\vartheta,\xi) + \alpha_t(\xi,h)\}\,w(\xi)\,d\xi$$

$$\rho_u(t,\xi,h) = \max_{u\in P(\xi)} \{h'(0)X(\vartheta,\xi) + \alpha_t(\xi,h)\}B(\xi)u$$

$$\rho_v(t,\xi,h) = \max_{v\in Q(\xi)} \{h'(0)X(\vartheta,\xi) + \alpha_t(\xi,h)\}C(\xi)v$$

$$\rho_N(h) = \min_{y\in N} \langle y, h\rangle$$

2. Let us note the properties of sets $W(t, N)$ needed subsequently. From (1.4) it is obvious that the set $W(t, N)$ is closed and convex in H for any $t \in [t_0, \vartheta]$. Set $W(t, N)$ is not bounded for $t \in [t_0, \vartheta)$ and $W(\vartheta, N) = N$ for $t = \vartheta$.

Lemma 2.1. The set $W(t_*, N)$ is weakly upper-semicontinuous for any $t_* \in [t_0, \vartheta]$, i.e. inclusion $y^*(s) \in W(t_*, N)$ is valid for any number sequence $\{t_k\}$ converging to t_* and any sequence $\{y^{(k)}(s)\}$, $y^{(k)}(s) \in W(t_k, N)$, weakly converging to $y^*(s) \in H$.

Proof. Assuming the contrary, we get that an instant $t_* \in [t_0, \vartheta]$, a sequence $\{t_k\}$ converging to t_* and a sequence $\{y^{(k)}(s)\}$, $y^{(k)}(s) \in W(t_k, N)$, weakly converging in H to some element $y^*(s)$ exist such that $y^*(s) \notin W(t_*, N)$. Let $v(t)$ be an arbitrary program. By definition a program $u_k(t)$ exists such that the function $x^{(k)}(t) = x(t; \{t_k, y^{(k)}(s)\}, u_k, v)$ satisfies the condition $x_\vartheta^{(k)}(s) \in N$. We now fix an arbitrary program $u(t)$ and we form the sequence $\{u_*^{(k)}(t)\}$ of functions

$$u_*^{(k)}(t) = \begin{cases} u(t), & t \in [t_0, t_k) \\ u_k(t), & t \in [t_k, \vartheta] \end{cases}$$

Without loss of generality we assume that $\{u_*^{(k)}(t)\}$ converges weakly (in L_2) to some program $u_*(t)$, $u_*(t) \in P(t)$ for almost all $t \in [t_0, \vartheta]$.

We shall show that the sequence $\{x^{(k)}(\vartheta + s)\}$ converges weakly in H to $x^*(\vartheta + s)$, where $x^*(t) = x(t; \{t_*, y^*(s)\}, u_*, v)$. This statement is obvious for $t_* \leqslant \vartheta - \tau$. Let $t_* > \vartheta - \tau$. In this case it suffices to show that

$$\Delta \equiv \int_{-\tau+\delta_k}^{0} h'(\xi - \delta_k)\, y^{(k)}(\xi)\, d\xi - \int_{-\tau+\delta_*}^{0} h'(\xi - \delta_*)\, y^*(s)\, ds \to 0 \quad \text{as} \quad k \to \infty$$

where

$$\delta_k = \begin{cases} \vartheta - t_k, & t_k \in [\vartheta - \tau, \vartheta] \\ \tau, & t_k < \vartheta - \tau \end{cases}$$

The following equality is valid:

$$\Delta = \int_{-\tau+\delta_*}^{0} h'(\xi - \delta_*)\{y^{(k)}(\xi) - y^*(\xi)\}\, d\xi + \int_{-\tau+\delta_*}^{0} \{h'(\xi - \delta_k) - h'(\xi - \delta_*)\}\, y^{(k)}(\xi)\, d\xi \tag{2.1}$$

Here the first integral in the right-hand side tends to zero since $\{y^{(k)}(s)\} \to y(s)$ weakly in H. Applying the Cauchy inequality and allowing as well for the boundedness of $\{y^{(k)}(s)\}$ and for the convergence of the integral

$$\int_{-\tau+\delta_*}^{0} \| h'(\xi - \delta_k) - h'(\xi - \delta_*)\|^2\, d\xi$$

to zero, we get that the second integral also is zero in the limit. Taking into account the weak closedness of set N, we have $x_\vartheta^*(s) \in N$. This contradicts the assumption because program $v(t)$ is arbitrary. The lemma is proved.

Let $y(s; h, t)$ be the element of $W(t, N)$, closest to $h = h(s)$ in H. The existence and uniqueness of such an element follows from the convexity and closedness of set $W(t, N)$ in H [8]. Following [4], we say that the system of sets $W(t, N)$, $t_0 \leqslant t \leqslant \vartheta$ is strongly u-stable if $W(t, N) \neq \phi$, $\forall t$ and the following condition is satisfied: for any $t_* \in [t_0, \vartheta)$, $t^* \in (t_*, \vartheta]$, $x_*(s) \in W(t, N)$ and program $v(t)$ a program $u(t)$ exists such that $x(t^* + s;\ \{t_*, x_*(s)\}, u, v) \in W(t^*, N)$.

Condition 2.1. The system of sets $W(t, N)$, $t_0 \leqslant t \leqslant \vartheta$, is strongly u-stable.

The validity of the next statement can be established by means of Lemma 2.1.

Lemma 2.2. Under Condition 2.1 the element $y(s; h, t)$ is weakly right-continuous in ϑ for any $h \in H$, i.e.

$$\lim_{\Delta t \to +0} \langle y(s;\ h,\ t+\Delta t),\ q(s)\rangle = \langle y(s;\ h,\ t),\ q(s)\rangle$$

for any instant $t \in [t_0,\ \vartheta]$ and any element $q(s) \in H$.

Definition [3]. A function $U_e(t,\ x(s))$ of the form

$$U_e(t,\ x(s)) = \{u_e\ (y(0;\ x(s),\ t) - x(0))\ B(t)\, u_e = \max_{u \in P(t)} (y(0;\ x(s),\ t) - x(0))\ B(t)\, u$$

is called an extremal strategy U_e.

Relying on Lemmas 2.1 and 2.2, we can verify the validity of

Theorem 2.1. Under Condition 2.1 the strategy $U_e(t,\ x(s))$ is admissible.

From Lemma 5 in [6] and Theorem 2.1 there obviously follows

Theorem 2.2. Let the initial position be such that $x_0(s) \in W(t,\ N)$. Under Condition 2.1 the extremal strategy U_e solves Problem 1 and, consequently, by virtue of the structure of the sets $W(t,\ N)$, solves Problem 2.

3. We indicate a method for finding the element $y(s;\ x,\ t)$. On the basis of Theorem 1.1, the weak continuity in H of the functional $\gamma(\vartheta,\ t,\ x)$ (1.4) and the minimax theorem [9] we can prove

Lemma 3.1. Let the functional $\rho(\vartheta, t, h)$ be convex in h and let $W(t,\ N) \neq \phi$. $W(t,\ N) \cap S_r(x(s)) \neq \phi$ if and only if the inequality

$$\max_{\|h\|_\tau \leqslant 1} \{-\rho(\vartheta, t, h) - \max_{y \in S_r(x(s))} \langle B(t, \vartheta, h, x\rangle\} \leqslant 0 \tag{3.1}$$

is satisfied. Here $S_r(x(s))$ is a closed sphere in H with center at $x(s)$ and radius r.

Inequality (3.1) is equivalent to the inequality

$$\max_{h \in F_c(t)} \{-\rho(\vartheta,\ t,\ h) - \langle B(t,\ \vartheta)\, h,\ x\rangle - r\|B(t,\ \vartheta)\, h\|_\tau\} \leqslant 0 \tag{3.2}$$

$$F_c(t) = \{h \in H \mid \|B(t,\ \vartheta)\, h\|_\tau \leqslant 1,\ \|h\|_\tau \leqslant c\}$$

Because the functional to be maximized in (3.2) is weakly continuous in h and the set $F_c(t)$ is weakly compact, the maximum in (3.2) is achievable. It cannot be achieved on the element h_*, $\|B(t,\ \vartheta)\, h_*\|_\tau = 0$, since then (1.4) is satisfied, which contradicts the condition $x(s) \notin W(t,\ N)$. In such a case (3.1) is equivalent to

$$\mu_1(\vartheta,\ t,\ x) = \max_{h \in F(t)} \{-\rho(\vartheta,\ t,\ h) - \langle B(t,\ \vartheta)\, h,\ x\rangle - r\|B(t,\ \vartheta)\, h\|_\tau\} \leqslant 0 \tag{3.3}$$

$$F(t) = \{h \in H \mid \ \|B(t,\ \vartheta)\, h\|_\tau = 1,\ \ \|h\|_\tau \leqslant 1\}$$

The smallest value of $r \geqslant 0$ for which (3.3) is satisfied, is exactly the distance from $x(s)$ to $W(t,\ N)$

$$r_0 = \begin{cases} \mu_1(\vartheta, t, x), & \mu_1(\vartheta, t, x) > 0 \\ 0, & \mu_1(\vartheta, t, x) \leqslant 0 \end{cases}$$

Let $h_0 = h_0(t, \vartheta,\ x)$ be the element of space H on which the maximum in (3.3) is achieved (the uniqueness of $B(t,\ \vartheta)\, h_0$ follows from the uniqueness of the element closest to $x(s)$ in $W(t, N)$). It is clear that the element $y(s; x(s), t)$ satisfies the maximum condition

$$\langle B(t, \vartheta) h_0, y(s; x, t)\rangle = \max_{y \in S_{r_0}(x(s))} \langle B(t, \vartheta) h_0, y\rangle$$

and, consequently, equals $r_0 B(t, \vartheta) h_0 + x(s)$. Thus, there holds the

Theorem 3.1. For $t = t_1$ let $W(t_1, N) \neq \phi$ and $x(s) \notin W(t_1, N)$ and let the functional $\rho(\vartheta, t_1, h)$ be convex in h ; then

$$y(s; x, t_1) = r_0 B(t_1, \vartheta) h_0 + x(s)$$

4. We indicate the sufficient conditions for the nonemptiness and strong u-stability of sets $W(t, N)$. We introduce the sets

$$T(t, x) = \{h_0 \in H \mid \|h_0\|_\tau \leqslant 1, \quad \gamma(\vartheta, t, x) = -\varphi(\vartheta, t, h_0, x)\}$$
$$D_l(K) = \{t \in [t_0, \vartheta], \quad x \in H \mid \|x\|_\tau \leqslant K, \quad \gamma(\vartheta, t, x) \geqslant l\}$$

It can be verified that the sets $T(t, x)$ and $D_l(K)$ are weakly closed in space H and $T(t, x)$ is weakly upper-semicontinuous in t and x.

Condition 4.1. For any vector $v^* \in Q(t)$ we can find a vector $u^* \in P(t)$ in the region $\gamma(\vartheta, t, x) > 0$ such that the inequality

$$\psi(t, u^*, v^*, h) \leqslant 0$$

is satisfied simultaneously for all elements $h \in T(t, x)$. Here

$$\psi(t, u, v, h) = \rho_u(t, t, h) - \rho_v(t, t, h) + \{h'(0) \times X(\vartheta, t) + \alpha_t(t, h)\} C(t) v - \{h'(0) X(\vartheta, t) + \alpha_t(t, h)\} B(t) u$$

We consider the system of sets (G is some finite set from H)

$$T(t_*, x_*, \eta, G) = \bigcup_{\{t,x\}} T(t, x)$$
$$|t - t_*| \leqslant \eta, \quad |\langle x - x_*, q_i\rangle| \leqslant \eta, \quad q_i \in G$$

Lemma 4.1. Let Condition 4.1 be fulfilled; then for any number $\alpha > 0$ and a finite set $G \subset H$ we can find a number $\eta(\alpha, G) > 0$ and a vector $u^* \in P(t_*)$ such that the inequality

$$\max_{v \in Q(t_*)} \max_{h \in T(\eta)} \psi(t_*, u^*, v, h) \leqslant \alpha$$
$$T(\eta) = T(t_*, x_*, \eta, G)$$

holds in the domain $D_l(K)$.

The lemma's proof follows from Condition 4.1, the continuity of functional $\psi(t, u, v, h)$ and the weak upper-semicontinuity in t and x of the sets $T(t, x)$.

Relying on Lemma 4.1, we can verify the validity of

Lemma 4.2. Let Condition 4.1 be satisfied. Then for a number $\alpha > 0$ we can find a number $\eta(\alpha) > 0$ such that for any program $v_*(t)$ we can find a program $u_*(t)$ ensuring the inequality

$$\gamma(\vartheta, t_* + \delta, x(t_* + \delta + s; \{t_*, x_*\}, u_*, v_*)) \leqslant \gamma(\vartheta, t_*, x_*) + \alpha\delta + \lambda\delta^2$$
$$\lambda = \sup\{\psi(t, u, v, h) \mid t \in [t_0, \vartheta], \ u \in P(t), \ v \in Q(t), \|h\|_\tau \leqslant 1\}$$

with $\delta \leqslant$ for any position $\{t_*, x_*(s)\} \in D_l(K)$.

By the symbol $W_\varepsilon(t, N)$ we denote the set of those and only those elements $x(s) \in H$ which satisfy the inequality $\gamma(\vartheta, t, x) \leqslant \varepsilon$.

Lemma 4.3. Let Condition 4.1 be satisfied and let $W_{\varepsilon_*}(t_0, N) \neq \phi$ for some $\varepsilon = \varepsilon_* > 0$. Then the sets $W_{\varepsilon_*}(t, N)$ are nonempty for all $t \in [t_0, \vartheta]$ and are strongly u-stable.

As a matter of fact, let there be given an arbitrary instant $t_1 \in [t_0, \vartheta]$, an element $q(s) \in W_{\varepsilon_*}(t_1, N)$, a number $\delta \in (0, \vartheta - t_1]$, a program $v_1(t)$ and a sequence of numbers $\beta_i \to +0$. By means of Lemma 4.2 we can show the existence of programs $u_i(t)$ such that

$$\gamma(\vartheta, t_1 + \delta, x(t_1 + \delta + s; \{t_1, q\}, u_i, v_1)) \leqslant \gamma(\vartheta, t_1, q) + \beta_i \delta$$

$u_i \to u_0$ weakly in L_2 as $i \to \infty$, $x(t_1 + \delta + s; \{t_1, q\}, u_i, v_i) \to x(t_1 + \delta + s; \{t_1, q\}, u_0, v_1)$ in H. Thus,

$$\gamma(\vartheta, t_1 + \delta, x(t_1 + \delta + s; \{t_1, q\}; u_i, v_1)) \leqslant \varepsilon_*$$

The nonemptiness of sets $W_{\varepsilon_*}(t, N)$ is proved similarly.

Using Lemma 4.3 and the proof plan for Theorem 2.2 from [5], we can verify the validity of

Theorem 4.1. If $W(t_0, N) \neq \phi$ and Condition 4.1 is satisfied, then the sets $W(t, N)$ are nonempty for all $t \in [t_0, \vartheta]$ and are strongly u-stable.

Corollary 4.1. Let $W(t_0, N) \neq \phi$ and let the maximum in (1.4) with $\gamma(\vartheta, t, x) > 0$ be achieved on a unique h; then the sets $W(t, N)$ are nonempty for $t \in [t_0, \vartheta]$ and are strongly u-stable.

Corollary 4.2. Let $W(t_0, N) \neq \phi$ and let the functional $\rho(\vartheta, t, h)$ be convex in h; then the sets $W(t, N)$ are nonempty for all $t \in [t_0, \vartheta]$ and are strongly u-stable.

From Theorems 2.1, 2.2 and 4.1 follows

Theorem 4.2. Let Condition 4.1 (or the hypotheses of Corollaries 4.1 and 4.2) be satisfied and let $\gamma(\vartheta, t_0, x_0) \leqslant 0$. Then the strategy U_e extremal to sets $W(t, N)$ solves Problems 1 and 2.

Note 4.1. From the statement of Problem 2 the first player constructs his own control for $t \geqslant \vartheta - \tau$ on the basis of some previous history of the system's motion. The question arises to what extent the knowledge of this previous history is necessary for the solvability of Problem 2. In other words, under what conditions can Problem 2 be solved in the class of strategies $U(t, x)$, where $x \in E_r$ and the set $U(t, x)$ is once again bounded, convex and upper-semicontinuous with respect to inclusion in t (from the right) and x. It can be shown that Problem 2 has a solution in the class of strategies $U(t, x)$, $x \in E_n$ if and only if there exists a system of sets $W_1(t)$, $\vartheta - \tau \leqslant t \leqslant \vartheta$, convex, closed and upper-semicontinuous in t, strongly u-stable in the sense of [1], such that for any function $\varphi(t) \in H$, $\varphi(t) \in W_1(t)$, the inclusion $\varphi(t) \in N$ is valid for almost all $t \in [\vartheta - \tau, \vartheta]$.

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ON THE PRECESSION EQUATIONS OF GYROSCOPIC SYSTEMS

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The analysis of equations of gyroscopic systems almost always necessitates the separation of fast nutation motions from those of slow precession. Simplified equations for the determination of these two kinds of motion can be obtained by various means [1]. The method of fractional analysis [2] used here for formalizing the passing to precession equations is based on the combination of methods of the theory of similarity and dimensionality with asymptotic methods of the theory of differential equations. The asymptotic behavior of solutions of complete equations of the gyroscopic system motions is investigated in the case in which the ratio of characteristic times T_n and T_p of nutation and precession motion components tends to zero.

Definition. Equations whose solutions for the slow components of motion represent for times of order T_p the zero order approximation with respect to the small parameter $\mu = T_n / T_p$, where T_n and T_p are the characteristic times of nutation and precession components, respectively, are called precession equations of gyroscopic systems. The exact meaning of this definition is made clear subsequently.

Let us briefly consider the problem of passing to precession equations. Some of such problems were considered earlier in [3, 4].

The general equations of gyroscopic systems are of the form [1]: